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# Calculus

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Calculus is the mathematical study of change, in the same way that geometry is the study of shape and algebra is the study of operations and their application to solving equations. It has two major branches, differential calculus (concerning rates of change and slopes of curves), [1] and integral calculus (concerning accumulation of quantities and the areas under and between curves); [2] these two branches are related to each other by the fundamental theorem of calculus. Both branches make use of the fundamental notions of convergence of infinite sequences and infinite series to a well-defined limit. Generally, modern calculus is considered to have been developed in the 17th century by Isaac Newton and Gottfried Leibniz. Today, calculus has widespread uses in science, engineering and economics[3] and can solve many problems that algebra alone cannot.

Calculus is a part of modern mathematics education. A course in calculus is a gateway to other, more advanced courses in mathematics devoted to the study of functions and limits, broadly called mathematical analysis. Calculus has historically been called "the calculus of infinitesimals", or "infinitesimal calculus". The word "calculus" comes from Latin (calculus) and refers to a small stone used for counting. More generally, calculus (plural calculi) refers to any method or system of calculation guided by the symbolic manipulation of expressions. Some examples of other well-known calculi are propositional calculus, calculus of variations, lambda calculus, and process calculus.

## History

Modern calculus was developed in 17th century Europe by Isaac Newton and Gottfried Wilhelm Leibniz (see the Leibniz–Newton calculus controversy), but elements of it have appeared in ancient Greece, China, medieval Europe, India, and the Middle East.

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## Ancient

The ancient period introduced some of the ideas that led to integral calculus, but does not seem to have developed these ideas in a rigorous and systematic way. Calculations of volume and area, one goal of integral calculus, can be found in the Egyptian Moscow papyrus (c. 1820 BC), but the formulas are simple instructions, with no indication as to method, and some of them lack major components. From the age of Greek mathematics, Eudoxus (c.408-355 BC) used the method of exhaustion, which foreshadows the concept of the limit, to calculate areas and volumes, while Archimedes (c. 287-212 BC) developed this idea further, inventing heuristics which resemble the methods of integral calculus. The method of exhaustion was later reinvented in China by Liu Hui in the 3rd century AD in order to find the area of a circle. In the 5th century AD, Zu Chongzhi established a method that would later be called Cavalieri's principle to find the volume of a sphere.

## Medieval

Alexander the Great's invasion of northern India brought Greek Trigonometry, using the chord, to India where the sine, cosine, and tangent were conceived[citation needed]. Indian mathematicians gave a semi-rigorous method of differentiation of some trigonometric functions. In the Middle East, Alhazen derived a formula for the sum of fourth powers. He used the results to carry out what would now be called an integration, where the formulas for the sums of integral squares and fourth powers allowed him to calculate the volume of a paraboloid. In the 14th century, Indian mathematician Madhava of Sangamagrama and the Kerala school of astronomy and mathematics stated components of calculus such as the Taylor series and infinite series approximations. However, they were not able to "combine many differing ideas under the two unifying themes of the derivative and the integral, show the connection between the two, and turn calculus into the great problem-solving tool we have today".

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## Modern

In Europe, the foundational work was a treatise due to Bonaventura Cavalieri, who argued that volumes and areas should be computed as the sums of the volumes and areas of infinitesimally thin cross-sections. The ideas were similar to Archimedes' in The Method, but this treatise is believed to have been lost in the 13th century, and was only rediscovered in the early 20th century, and so would have been unknown to Cavalieri. Cavalieri's work was not well respected since his methods could lead to erroneous results, and the infinitesimal quantities he introduced were disreputable at first.

The formal study of calculus brought together Cavalieri's infinitesimals with the calculus of finite differences developed in Europe at around the same time. Pierre de Fermat, claiming that he borrowed from Diophantus, introduced the concept of adequality, which represented equality up to an infinitesimal error term. The combination was achieved by John Wallis, Isaac Barrow, and James Gregory, the latter two proving the second fundamental theorem of calculus around 1670.

The product rule and chain rule, the notion of higher derivatives, Taylor series, and analytical functions were introduced by Isaac Newton in an idiosyncratic notation which he used to solve problems of mathematical physics. In his works, Newton rephrased his ideas to suit the mathematical idiom of the time, replacing calculations with infinitesimals by equivalent geometrical arguments which were considered beyond reproach. He used the methods of calculus to solve the problem of planetary motion, the shape of the surface of a rotating fluid, the oblateness of the earth, the motion of a weight sliding on a cycloid, and many other problems discussed in his Principia Mathematica (1687). In other work, he developed series expansions for functions, including fractional and irrational powers, and it was clear that he understood the principles of the Taylor series. He did not publish all these discoveries, and at this time infinitesimal methods were still considered disreputable.

These ideas were arranged into a true calculus of infinitesimals by Gottfried Wilhelm Leibniz, who was originally accused of plagiarism by Newton. He is now regarded as an independent inventor of and contributor to calculus. His contribution was to provide a clear set of rules for working with infinitesimal quantities, allowing the computation of

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second and higher derivatives, and providing the product rule and chain rule, in their differential and integral forms. Unlike Newton, Leibniz paid a lot of attention to the formalism, often spending days determining appropriate symbols for concepts.

Leibniz and Newton are usually both credited with the invention of calculus. Newton was the first to apply calculus to general physics and Leibniz developed much of the notation used in calculus today. The basic insights that both Newton and Leibniz provided were the laws of differentiation and integration, second and higher derivatives, and the notion of an approximating polynomial series. By Newton's time, the fundamental theorem of calculus was known.

When Newton and Leibniz first published their results, there was great controversy over which mathematician (and therefore which country) deserved credit. Newton derived his results first (later to be published in his Method of Fluxions), but Leibniz published his Nova Methodus pro Maximis et Minimis first. Newton claimed Leibniz stole ideas from his unpublished notes, which Newton had shared with a few members of the Royal Society. This controversy divided English-speaking mathematicians from continental mathematicians for many years, to the detriment of English mathematics. A careful examination of the papers of Leibniz and Newton shows that they arrived at their results independently, with Leibniz starting first with integration and Newton with differentiation. Today, both Newton and Leibniz are given credit for developing calculus independently. It is Leibniz, however, who gave the new discipline its name. Newton called his calculus "the science of fluxions".

Since the time of Leibniz and Newton, many mathematicians have contributed to the continuing development of calculus. One of the first and most complete works on finite and infinitesimal analysis was written in 1748 by Maria Gaetana Agnesi.

## Foundations

In calculus, foundations refers to the rigorous development of a subject from precise axioms and definitions. In early calculus the use of infinitesimal quantities was thought unrigorous, and was fiercely criticized by a number of authors, most notably Michel Rolle and Bishop

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Berkeley. Berkeley famously described infinitesimals as the ghosts of departed quantities in his book The Analyst in 1734. Working out a rigorous foundation for calculus occupied mathematicians for much of the century following Newton and Leibniz, and is still to some extent an active area of research today.

Several mathematicians, including Maclaurin, tried to prove the soundness of using infinitesimals, but it would not be until 150 years later when, due to the work of Cauchy and Weierstrass, a way was finally found to avoid mere "notions" of infinitely small quantities.[15] The foundations of differential and integral calculus had been laid. In Cauchy's writing (see Cours d'Analyse), we find a broad range of foundational approaches, including a definition of continuity in terms of infinitesimals, and a (somewhat imprecise) prototype of an (e, d)-definition of limit in the definition of differentiation. In his work Weierstrass formalized the concept of limit and eliminated infinitesimals. Following the work of Weierstrass, it eventually became common to base calculus on limits instead of infinitesimal quantities, though the subject is still occasionally called "infinitesimal calculus". Bernhard Riemann used these ideas to give a precise definition of the integral. It was also during this period that the ideas of calculus were generalized to Euclidean space and the complex plane.

In modern mathematics, the foundations of calculus are included in the field of real analysis, which contains full definitions and proofs of the theorems of calculus. The reach of calculus has also been greatly extended. Henri Lebesgue invented measure theory and used it to define integrals of all but the most pathological functions. Laurent Schwartz introduced distributions, which can be used to take the derivative of any function whatsoever.

Limits are not the only rigorous approach to the foundation of calculus. Another way is to use Abraham Robinson's non-standard analysis. Robinson's approach, developed in the 1960s, uses technical machinery from mathematical logic to augment the real number system with infinitesimal and infinite numbers, as in the original Newton-Leibniz conception. The resulting numbers are called hyperreal numbers, and they can be used to give a Leibniz-like development of the usual rules of calculus.

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## Significance

While many of the ideas of calculus had been developed earlier in Egypt, Greece, China, India, Iraq, Persia, and Japan, the use of calculus began in Europe, during the 17th century, when Isaac Newton and Gottfried Wilhelm Leibniz built on the work of earlier mathematicians to introduce its basic principles. The development of calculus was built on earlier concepts of instantaneous motion and area underneath curves.

Applications of differential calculus include computations involving velocity and acceleration, the slope of a curve, and optimization. Applications of integral calculus include computations involving area, volume, arc length, center of mass, work, and pressure. More advanced applications include power series and Fourier series.

Calculus is also used to gain a more precise understanding of the nature of space, time, and motion. For centuries, mathematicians and philosophers wrestled with paradoxes involving division by zero or sums of infinitely many numbers. These questions arise in the study of motion and area. The ancient Greek philosopher Zeno of Elea gave several famous examples of such paradoxes. Calculus provides tools, especially the limit and the infinite series, which resolve the paradoxes.

## Applications

Calculus is used in every branch of the physical sciences, actuarial science, computer science, statistics, engineering, economics, business, medicine, demography, and in other fields wherever a problem can be mathematically modeled and an optimal solution is desired. It allows one to go from (non-constant) rates of change to the total change or vice versa, and many times in studying a problem we know one and are trying to find the other.

Physics makes particular use of calculus; all concepts in classical mechanics and electromagnetism are related through calculus. The mass of an object of known density, the moment of inertia of objects, as well as the total energy of an object within a conservative field can be found by the use of calculus. An example of the use of calculus in mechanics is Newton's second law of motion: historically stated it expressly uses the term "rate of change" which refers to the derivative saying The rate of change of momentum of a body is equal to the resultant force acting on

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the body and is in the same direction. Commonly expressed today as Force = Mass × acceleration, it involves differential calculus because acceleration is the time derivative of velocity or second time derivative of trajectory or spatial position. Starting from knowing how an object is accelerating, we use calculus to derive its path.

Maxwell's theory of electromagnetism and Einstein's theory of general relativity are also expressed in the language of differential calculus. Chemistry also uses calculus in determining reaction rates and radioactive decay. In biology, population dynamics starts with reproduction and death rates to model population changes.

Calculus can be used in conjunction with other mathematical disciplines. For example, it can be used with linear algebra to find the "best fit" linear approximation for a set of points in a domain. Or it can be used in probability theory to determine the probability of a continuous random variable from an assumed density function. In analytic geometry, the study of graphs of functions, calculus is used to find high points and low points (maxima and minima), slope, concavity and inflection points.

Green's Theorem, which gives the relationship between a line integral around a simple closed curve C and a double integral over the plane region D bounded by C, is applied in an instrument known as a planimeter, which is used to calculate the area of a flat surface on a drawing. For example, it can be used to calculate the amount of area taken up by an irregularly shaped flower bed or swimming pool when designing the layout of a piece of property.

Discrete Green's Theorem, which gives the relationship between a double integral of a function around a simple closed rectangular curve C and a linear combination of the antiderivative's values at corner points along the edge of the curve, allows fast calculation of sums of values in rectangular domains. For example, it can be used to efficiently calculate sums of rectangular domains in images, in order to rapidly extract features and detect object - see also the summed area table algorithm.

In the realm of medicine, calculus can be used to find the optimal branching angle of a blood vessel so as to maximize flow. From the decay laws for a particular drug's elimination from the body, it is used to derive

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dosing laws. In nuclear medicine, it is used to build models of radiation transport in targeted tumor therapies.

In economics, calculus allows for the determination of maximal profit by providing a way to easily calculate both marginal cost and marginal revenue.

Calculus is also used to find approximate solutions to equations; in practice it is the standard way to solve differential equations and do root finding in most applications. Examples are methods such as Newton's method, fixed point iteration, and linear approximation. For instance, spacecraft use a variation of the Euler method to approximate curved courses within zero gravity environments.

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